MANONMANIAM SUNDARANAR UNIVERSITY

M.Phil Mathematics (for Affiliated Colleges)

(From the academic year 2018 -19)

1. SCHEME OF EXAMINATION

Sl. No	Sem-ester	Paper title	Hrs/Week	Credits
1.	I	Research and Teaching Methodology	4	4
2.	I	Advanced Analysis	4	4
3	I	Project Oriented Elective Course (Theory)	4	4
4	II	Project and Viva Voce		12
		TOTAL		24

LIST OF PROJECT ORIENTED ELECTIVE PAPERS

1 Banach Algebra and Spectral Theory
2 Advanced Graph Theory
3 Harmonic Analysis
4 Theory of Near-rings
5 Advanced Calculus
6 Algebraic Graph Theory
7 Stochastic Modeling.
8. Wavelets

Core Paper- I

RESEARCH AND TEACHING METHODOLOGY (60 hours)

It is the study of commutative rings. The objective of the paper is to introduce algebraic structure through the modules and different types of modules and it's algebraic application. A pass in PG level algebra course is the prerequisite for this paper. Out come of this paper is to motivate students to do research in diverse fields such as homological algebra, algebraic number theory, algebraic geometry, finite fields and computational algebra.

Unit I: Rings and Ideals – Modules **(12 hours)**

Unit II: Rings and Modules fractions – Primary Decomposition **(12 hours)**

Unit III: Integral Dependence and valuations – Chain conditions **(12 hours)**

Unit IV: Noetherian Rings – Artin Rings **(12 hours)**

Unit V: Methodology of Teaching **(12 hours)**

Teaching - Objectives of Teaching, Phases of Teaching - Teaching Methods: Lecture Method, Discussion Method, Discovery Learning, Inquiry, Problem Solving Method, Project Method, Seminar - Integrating ICT in Teaching: Individual Instruction, Ways for Effective Presentation with Powerpoint - Documentation - Evaluation: Formative, Summative & Continuous and Comprehensive Evaluation - Later Adolescent Psychology: Meaning, Physical, Cognitive, Emotional, Social and Moral Development - Teaching Later Adolescents.

Text Book: Content and Treatment as in Atiyah and Macdonald, Introduction to Commutative Algebra, Chapters 1 to 9.

References:

1. Sampath, K., Pannerselvam, A. & Santhanam, S. (1984). Introduction to educational technolog. (2nd revised ed.). New Delhi: Sterling Publishers.
2. Sharma, S. R. (2003). Effective classroom teaching modern methods, tools & techniques. Jaipur: Mangal Deep.
3. Vedanayagam, E. G. (1989). Teaching technology for college teachers. New York: Sterling Publishers.

Core Paper II

ADVANCED ANALYSIS (60 hours)

Preamble: The objective of the paper is to understand borel measure in real and complex. Field. Prerequisite for this course is a good knowledge in calculus, real and complex analysis, topology and measure theory concepts. Motivation is to prepare scholars with Lp spaces for the study of analysis.. The out come of this paper is to help the students to undertake further research in Fourier analysis, Harmonic analysis and Functional analysis.

Unit I : Abstract Integration : The concept of measurability – Simple functions – Elementary properties of measures – Arithmetic in $[0, \infty]$ - Integration of positive functions – Integration of complex functions – The role played by sets of measure zero. **(12 hours)**

Unit II : Positive Boral Measures : Topological preliminaries – The Riesz representation theorem – Regularity properties of Borel measures – Lebesgue measure – Continuity properties of measurable functions. **(12 hours)**

Unit III : Complex Measures : Total variation – Absolute continuity – Consequences of the Radon-Nikodym theorem – Bounded linear functions on L^p- The Riesz representation theorem. **(12 hours)**
Unit IV : H^p- Spaces : Sub-harmonic functions – The spaces H^p and N - The theorem of F. and M. Reisz – Factorization theorems – The shift operator – Conjugate functions. **(12 hours)**

Unit V : Fourier Transforms : Formal properties – The inversion theorem – The Plancherel theorem – The Banach algebra L^1.
Holomorphic Fourier Transforms : Two theorems of Paley and Wiener – Quasi-analytic classes – The Denjoy- Careman theorem. **(12 hours)**

Text Book : Content and Treatment as in Walter Rudin, Real and Complex Analysis, Third Edition, Chapters 1, 2, 6, 9, 17 and 19.

PAPER III

PROJECT ORIENTED ELECTIVE COURSE (THEORY)

1. BANACH ALGEBRA AND SPECTRAL THEORY (60 hours)

Preamble: This syllabus is designed to introduce the students to the topics of Banach algebra and Hilbert spaces. Knowledge expected is to be aware of the background concepts in algebra. The students are expected to know about functionals. This will motivate the students to learn about various operators and their characteristics.

Unit I: Banach algebras – Complex Homomorphisms – Basic properties of Spectra – Symbolic Calculus. **(12 hours)**

Unit II: Differentiation - Group of invertible elements – Commutative Banach algebra – Ideals and Homomorphisms – Gelfand transforms. **(12 hours)**

Unit III: Involutions – Applications to non commutative algebra – Positive Linear functionals. **(12 hours)**

Unit IV: Bounded Operators on Hilbert spaces – Bounded Operators – A commutativity theorem – Resolution of the Identity – Spectral theorem. **(12 hours)**

Unit V: Eigen values of normal operators – Positive operators and square roots – Group of invertible operators – Characterization of V* algebra. **(12 hours)**

Text Book: Content and Treatment as in Rudin, Functional Analysis, Tata McGraw Hill, Chapters 10,11 & 12.

2. ADVANCED GRAPH THEORY (60 hours)

Preamble: This course aims to introduce the learner some topics for his research in graph theory. It provides several conjectures and open problems to widen the scope of research. The pre-requisite for the course is a sound knowledge in graph theory at the post-graduate level. The outcome of the course is identification area and problems for research in graph theory.

Unit I: Dominating sets in graphs - Bounds on the domination number: in terms of order, degree, size, degree, diameter and girth. **(12 hours)**

Unit II: Product graphs and Vizing's conjecture – Domatic number - Nordhaus-Gaddum type theorems - dominating functions. **(12 hours)**

Unit III: Decompositions and colorings of a graph – Generalizations of graph decompositions. **(12 hours)**

Unit IV: Necessary conditions for the existence of a G-decomposition of a graph - Cycle decompositions, Vertex labelings and graceful graphs. **(12 hours)**

Unit V: Perfect graphs: The perfect graph theorem – p-critical and partitionable graphs – A polyhedral characterization of perfect graphs and p-critical graphs – The strong perfect graph conjecture (and recent theorem). **(12 hours)**

Text Books: Content and Treatment as in

1) Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater, Fundamentals of Domination in graphs, Marcel Decker (1998), Section 1.2, 2.1to2.4 (For Unit I)
   Sections 2.6, 8.3, 9.1 and 10.1 to10.3 (for Unit II)

2) Juraj Bosak, Decompositions of graphs , Kluwar Academic Publishers, Chapters 2, 3 4, 6 and 7. (for Units III and IV)

3) Martin Charles Golumbic, Algorithmic graph theory, Academic Press, Chapter 3 (for Unit V)

3. HARMONIC ANALYSIS (60 hours)

Preamble: Periodic functions play a vital role in solving many problems in Mathematics and Physics. Fourier analysis is the study of various aspects of periodicity of functions. Harmonic Analysis is a natural generalization of Fourier analysis and is significant for its mathematical aspect. The pre requisite for this course is a basic knowledge of Real and Complex analysis covered in a post graduate programme in Mathematics. The outcome of the course is to help researchers in both pure and applied mathematical fields.

Unit I: Fourier series and integrals – Definitions and easy results – The Fourier transform – Convolution – Approximate identities – Fejer’s theorem – Unicity theorem – Parselval relation – Fourier Stieltjes Coefficients – The classical kernels. **(12 hours)**

Unit II: Summability – Metric theorems – Pointwise summability – Positive definite sequences – Herglotz;s theorem – The inequality of Hausdorff and Young. **(12 hours)**

Unit III: The Fourier integral – Kernels on R. The Plancherel theorem – Another convergence theorem – Poisson summation formula – Bachner’s theorem – Continuity theorem. **(12 hours)**

Unit IV: Characters of discrete groups and compact groups – Bochners’ theorem – Minkowski’s theorem. **(12 hours)**

Unit V: Hardy spaces- Invariant subspaces – Factoring F and M. Rieza theorem – Theorems of Szego and Beuoling. **(12 hours)**

Text Book: Content and Treatment as in Henry Helson, Harmonic Analysis, Hindustan Book Agency, Chapters 1.1 to 1.9, 2.1 to 3.5 and 4.1 to 4.3.

4. THEORY OF NEAR-RINGS (60 hours)

Preamble: The main objective of this course is to provide the knowledge about the generalized ring structures. In fact, near-ring is a natural generalization of rings in the sense that the set of all endomorphisms of a group form a ring, where the set of all mappings of a group form a near-ring. The structure of near-rings is useful in project geometry to deal about generalized field conditions.

Unit I: The elements of theory of near-rings. **(12 hours)**

Unit II: Ideal theory. **(12 hours)**

Unit III: Elements of structure theory. **(12 hours)**

Unit IV: Near-fields. **(12 hours)**

Unit V: More classes of near-rings. **(12 hours)**

Text Book: Content and Treatment as in G. Pilz, Theory of Near-rings, North Holland, Chapters 1,2,3, 8(a), 9(a) and 9(b).

5. ADVANCED CALCULUS (60 hours)

Preamble: The Calculus of several variables involves many branches of Mathematics such as Partial Differential Equations, Optimization, Statistics etc. The main objective of this course is to give a thorough understanding of differentiation and integration of functions of several variables. The prerequisite is a precise knowledge of Calculus of single variable. The outcome of the course is the ability to solve problems involving several variables.

Unit I : Differentiation – Basic theorems – Partial derivatives – Derivatives – Inverse functions. **(12 hours)**

Unit II : Implicit functions – Integration – Measure zero and Content zero – Integrable functions. **(12 hours)**

Unit III : Fubini's theorem – Partitions of Unity – Change of Variables. **(12 hours)**

Unit IV : Integration on chains - Algebraic preliminaries – Fields and Forms - Geometric preliminaries – The fundamental theorem of Calculus. **(12 hours)**

Unit V : Manifolds – Fields and Forms on Manifolds – Stokes' theorem on Manifolds - The Volume element – The Classical theorems. **(12 hours)**

Text book : Calculus on Manifolds by Michael Spivak, The Benjamin / Cummings Publishing Company. **(12 hours)**

References :
(1) Mathematical Analysis by Tom M. Apostol, Narosa Publishing Company.
(2) Advanced Calculus by Gerald B.Folland, Pearson Publishing Company.

6. ALGEBRAIC GRAPH THEORY (60 hours)

Preamble: This course aims to improve the knowledge of the learner to apply algebra in graph theory. It is framed to give adequate exposure about algebraic approach to graph theory. The beginner of this course is expected to have sound understanding of graph theory and algebra at PG level. The outcome of the course is to enable the student to do qualitative research in algebraic graph theory.

Unit 1: Linear Algebra in graph theory: The spectrum of a graph – Regular graphs and line graphs - The homology of graphs. **(12 hours)**

Unit 2: Spanning trees and associated structures – Complexity – Determinant expansions. **(12 hours)**

Unit 3: Symmetry and regularity of graphs: General properties of graph automorphisms – Vertex-transitive graphs – Symmetric graphs – Trivalent symmetric graphs. **(12 hours)**

Unit 4: The Covering - graph construction – Distance-transitive graphs - The feasibility of intersection arrays. **(12 hours)**

Unit 5: The Laplacian of a graph: The Laplacian matrix – trees – representations – energy and eigenvalues – connectivity – the generalized Laplacian – Multiplicities – embedding. **(12 hours)**

Text Books:

1. **Norman Biggs,** Algebraic Graph Theory, Cambridge University Press, London, 1974. Chapters 2, 3 and 4 for Unit I, 5, 6 and 7 for Unit II,C 15, 16, 17 and 18 for Unit III,19, 20 and 21 for Unit IV.

2. **Chris Godsil, Gordon Royle,** Algebraic Graph Theory, Springer-Verlag, New York, 2006. Chapter 13 (Sections 13.1 to 13.6, 13.9 to 13.11) for Unit V.

7. STOCHASTIC MODELING (60 hours)

Preamble: The theory of stochastic modelling is considered to be an important contribution to mathematics and it is an active topic of research. It is concerned with concepts and techniques and it is oriented towards a broad spectrum of mathematical, scientific and engineering interests. Characterization, structural properties , inferences and control of Stochastic processes are covered in every unit. The paper is designed to get deep knowledge of stochastic processes.

Recap : Basics of Probability space random variable – Discrete distributions and Continuous distributions – Expectation – Conditional Expectation – Moment Generating Function – Probability Generating Function – Laplace Transform – Joint Distributions – Functions of random variables and random vectors.

Unit I : Markov chains : Transition probability matrix of a Markov chain – First step Analysis – Functional of Random walks and successive runs – classification of states – Basic Limit Theorem of Markov Chain. **(12 hours)**

Unit II : Continuous time Markov Chains : Poisson distribution and Poisson process – Distributions associated with Poisson process – Pure Birth Process – Pure Death process – Birth and Death Process – Limiting behavior of Birth and Death Process – Birth and Death Process with absorbing states. **(12 hours)**

Unit III : Renewal Phenomena : Renewal process and Related concepts – Poisson process viewed a Renewal Process – Asymptotic behavior of Renewal process. **(12 hours)**

Unit IV : Branching Process and Population Growth : Branching process – branching process and generating functions – Geometrically distributed offspring – variation on Branching process – Stochastic models of Plasmid Reproduction and Plasmid copy Number partition. **(12 hours)**

Unit V : Queueing Systems : Queueing Processes – Poisson Arrival and exponentially distributed service times – The M/G/I and M/G/8 systems – variations and extensions. **(12 hours)**

Text Book : Content and Treatment as in Howard M. Taylor and Samuel Karlin, An Introduction to Stochastic Modeling (Revised Version), Academic Press, New York, 1984.

8. WAVELETS (60 hours)

Preamble: Wavelet analysis has drawn much attention from both mathematicians and engineers alike. The emphasis of the course is on spline wavelets and time-frequency analysis. The only pre-requisite is a basic knowledge of function theory and real analysis. The outcome of the course is to enable the learner to apply the pure mathematics in signal processing and image analysis.

Unit I : An Overview : Fourier to Wavelets – Integral Wavelets Transform and Time frequency analysis – Inversion formulas and duals – Classification of Wavelets – Multi-resolution analysis – Spines and Wavelets.
Fourier Analysis : Fourier and Inverse Fourier Transformation – Continuous Time Convolution – The delta function – Fourier Transformation of square integrable functions. **(12 hours)**

Unit II : Fourier Analysis (contd): Fourier Series – Basic Convergence Theory – Poisson Summation Formula.
Wavelet Transforms and Time Frequency Analysis : The Gabor Transforms – Short time Fourier Transforms and the uncertainty principle – The integral Wavelet Transform – Dyadic Wavelets – Inversion – Frames – Wavelet Series. **(12 hours)**

Unit III : Cardinal Spline Analysis : Cardinal Spline spaces – B-splines and their basic properties – The time scale relation and an interpolating graphical display algorithm – B-Net representations and computation of cardinal splines - Constructions of cardinal splines – constructions of spline application formulas – Construction of Spline interpolation formulas. **(12 hours)**

Unit IV : Scaling functions and Wavelets : Multi-resolution analysis – Scaling functions with finite two scale relation – Direction sum Decompositions of $L^2(R)$ - Wavelets and their duals. **(12 hours)**

Unit V : Cardinal Splines Wavelets : Interpolating splines wavelets – Compactly supported spline – Wavelets – Computation of Cardinal spline Wavelets – Euler – Frebenious Polynomials. **(12 hours)**
Orthogonal Wavelets : Examples of orthogonal Wavelets - Identification of orthogonal two scale symbols - Construction of compactly supported orthogonal wavelets. **(12 hours)**

Text Book : Content and Treatment as in Charles K. Chui, An introduction to Wavelets, Academic Press, New York, 1992.

Reference Books :

1. Chui C. K. (ed), Approximation theory and Fourier Analysis, Academic Press Boston, 1991.
2. Daribechies I, Wavelets, CBMS-NSF Series in Appl, SIAM Philadelphia, 1992.
3. Schurnaker L, L. Spline Functions : Basic Theory, Wiley, New York, 1981.
4. Nurnberger G, Applications to Spline Functions, Springer Verlag, New York, 1989.

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